EXPERIMENTAL DETERMINATION OF THE INERTIA OF THERMAL BOUNDARY LAYERS

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The article shows that the inertia of thermal boundary layers manifests itself at relatively high frequencies which, however, are perfectly attainable in experiments, and that it represents a substantial correction of the usually determined thermal inertia of thermal gauges.

The formation of a signal of contact thermometers and thermoanemometers is largely determined by the characteristics of their thermal and hydrodynamic layers. In measurements of the pulsations of temperature and speed, the boundary layers are usually regarded as quasisteady, i.e., it is assumed that at each instant their characteristics are exactly the same as in steady-state boundary layers that have at this instant the instantaneous speeds and temperatures of the incoming stream. In other words, boundary layers are assumed to be inertialess. The consequence is that the problem of internal unsteady heat exchange of a gauge is solved with boundary conditions of the third kind with constant heat-transfer coefficient. With such an approximation the inertia of thermometers and thermoanemometers has been studied fairly well [1].

It is obvious that this approximation is valid at slow changes of speed and temperature, but not at rapid changes. There exist theoretical [2-4] and experimental investigations [4-6] to substantiate more accurately this general statement.

It was established that at high oscillation frequencies, the amplitude of the surface tangential stress increases in comparison with its quasisteady value, and the amplitude of the heat exchange decreases. Experimental verification was carried out essentially with film thermoanemometers of constant temperature at sinusoidal transposition of the gauges parallelly to the stream and with simultaneous distortion of the hydrodynamic and thermal boundary layers.

The present article examines the periodic distortions of the thermal boundary layer only, with a steady hydrodynamic boundary layer. Both boundary layers are considered to be plane.

The principal parameter determining the inertia of a laminar boundary layer can be determined from an analysis of the equation of nonsteady forced convection [2]:

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}-a\frac{\partial^2 t}{\partial y^2}=-\frac{\partial t}{\partial \tau}. \tag{1}$$

The inertia of the thermal boundary layer manifests itself when the terms of the left-hand side of Eq. (1) are commensurable with its right-hand side.

Let us examine the case when a hydrodynamic boundary layer forms on a plate around which a liquid flows longitudinally or on some other body with small longitudinal velocity gradient, and a thermal boundary layer forms on the heated part situated at some distance from the front edge (Fig. 1). In this case the order of magnitude of the first and third terms on the left-hand side of (1) is given by the relations

$$u\frac{\partial t}{\partial x}\approx\frac{U}{d};\quad a\frac{\partial^2 t}{\partial y^2}\approx\frac{a}{\delta_t^2}. \tag{2}$$

The second term has the same order of magnitude as the first one.

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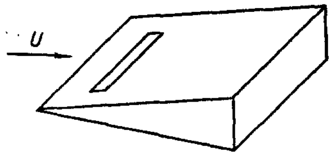

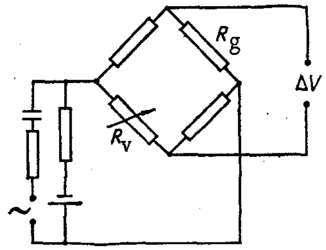

Fig. 1 Fig. 2

Fig. 1. Diagram of the disposition of a heated film in longitudinal flow around a plate.

Fig. 2. Diagram of the experimental determination of the AFC of a film gauge.

With sinusoidal oscillations of heat exchange the inertia due to the first two terms manifests itself on the frequencies at which

$$\frac{U}{d} \leqslant \omega, \quad \text{or} \quad \frac{\omega d}{U} \geqslant 1. \tag{3}$$

To estimate the third term, we determine the thickness of the thermal boundary layer. The ratio of the thicknesses of the thermal and the hydrodynamic boundary layers is given by the relation

$$\frac{\delta_t}{\delta} = \mathrm{Pr}^{-\frac{1}{3}} \left[1 - \left(\frac{x-d}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}. \tag{4}$$

When $d \ll x$, we obtain from (4) that

$$\frac{\delta_t}{\delta} = 0.91 \mathrm{Pr}^{-\frac{1}{3}} \left(\frac{d}{x}\right)^{\frac{1}{3}}. \tag{5}$$

The thickness of the hydrodynamic boundary layer is determined by the relation

$$\delta = 5 \sqrt{\frac{\nu x}{U}}. \tag{6}$$

From (5) and (6) we have

$$\delta_t = 4.5 \, \mathrm{Pr}^{-\frac{1}{3}} \left(\frac{d}{x}\right)^{\frac{1}{3}} \left(\frac{\nu x}{U}\right)^{\frac{1}{2}}. \tag{7}$$

If we substitute (7) into the second relation (2), we obtain the condition of the appearance of inertia due to the second terms:

$$\frac{\omega d}{U} \geqslant 0.05 \, \mathrm{Pr}^{-\frac{1}{3}} \left(\frac{d}{x}\right)^{\frac{1}{3}}. \tag{8}$$

The inequalities (3) and (8) indicate that the inertia of heat exchange is determined by the dimensionless parameter $\omega d/U$. Condition (3) indicates that inertia begins to manifest itself at the same frequencies at which local temporary changes of temperature occur as fast as the changes under the effect of convection. In quasisteady regime, temporary changes proceed much more slowly than changes due to transfer. Condition (8) describes the effect of thermal inertia of the boundary layer in heat propagation in it solely by heat conduction. The velocity of the stream then affects only the thickness of the boundary layer. This condition is satisfied at high frequencies when the speed of convective transfer along the surface may be neglected in comparison with the rate of the local change of temperature.

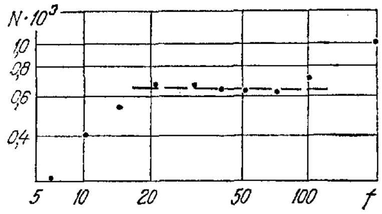

Fig. 3

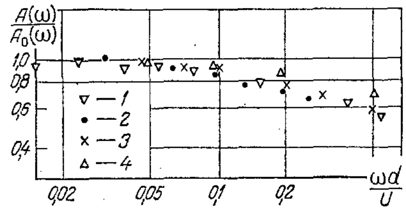

Fig. 4

Fig. 3. Change of the indicator N in dependence on the frequency. N, sec; f, Hz.

Fig. 4. Dimensionless AFC of the thermal boundary layer: 1) U = 0.65 m/sec; 2) 1.5; 3) 4.9; 4) 10.

The object of the experimental verification in the present work was to confirm that the inertia of heat exchange in the nonsteady boundary layer is in fact determined by the dimensionless parameter $\omega d/U$. It was carried out for the example of a thermoanemometric film gauge. The inertia of such a gauge is due to nonsteady conductive heat exchange inside it as well as to the nonsteadiness of the boundary layer.

When the boundary layer is nonsteady, the thermal inertia of a film gauge is theoretically calculated on the basis of the solution of the equation of nonsteady heat conduction. The amplitude–frequency curve (AFC) of the film gauge has the same shape as the AFC of the surface of the substrate [8]:

$$A(\omega) = \frac{1}{1 + \sqrt{2\omega N} + \omega N}. \tag{9}$$

The parameter N characterizing the thermal inertia of the gauge is determined by the intensity of the convective heat exchange, and in the case under consideration it does not depend on the frequency. With a nonsteady boundary layer, N will not be a constant any more.

The experiment was carried out in the following manner. The gauge was placed in a stream of water. While the film was heated by a sinusoidally changing electric current, the AFC was determined. From it we separated the part due to the inertia of the boundary layer. A diagram of the installation is shown in Fig. 2. The investigated gauge is placed as one arm in a balanced dc bridge and is heated by current to the specified operating temperature. The bridge is being balanced. The sinusoidally changing current with an amplitude amounting to not more than 10% of the dc is conducted to the same point of the bridge as the dc.

To determine the AFC, the frequency of the ac is changed while its amplitude is retained, and the voltage at the bridge output is recorded. From the obtained data the frequency dependence of the magnitude $|\Delta V(f)/\Delta V(0)|$ is plotted, i.e., the AFC.

On the basis of the determined experimental values of $A(\omega)$ and of relation (9) the values of N were found and the dependence $N = f(\omega)$, presented in Fig. 3, was plotted. This is due to the fact that the length of the thermal wave becomes commensurable with the width of the film, and the unidimensional model of heat exchange, on the basis of which relation (9) was obtained, is not valid. This divergence is not of great practical importance because the AFC in this region is close to 1. At medium frequencies N is constant, and relation (9) applies. The value of N then is equal to the length of the line segment intercepted by the dashed line on the axis of ordinates.

At high frequencies the experimentally determined value increases with increasing frequency. For these frequencies we plotted the dependence shown in Fig. 4, in the form

$$\frac{A(\omega)}{A_0(\omega)} = f\left(\frac{\omega x}{U}\right). \tag{10}$$

Here $A_0(\omega)$ denotes the values of the AFC corresponding to the range in which N is constant.

At different flow velocities all the data are grouped around one curve. That means that a deviation of N from the constant value at high frequencies is due to the inertia of the thermal boundary layer.

This deviation cannot be explained by the thermal inertia of a metallic sensitive film. Firstly, the thickness of the film does not exceed $0.5 \cdot 10^{-6}$ m, and the frequency is too low for its inertia to manifest itself. Secondly, the AFC of a thin film depends on the dimensionless parameter ωM with a time constant $M \sim 1/\alpha$. Since in laminar flow $\alpha \sim U^{+0.5}$, $M \sim \omega \cdot U^{-0.5}$, and the AFC is bound to depend on the parameter $\omega U^{-0.5}$, and not $\omega d/U$. In that case we would not obtain a single curve only in Fig. 4.

It follows that the inertia of the thermal boundary layer manifests itself at relatively high frequencies which, however, are perfectly attainable in experiments. This may introduce substantial corrections of the values of the inertia of the thermometer and thermoanemometer determined solely on the basis of the heat conduction inside the gauge and of the intensity of the steady heat exchange on its surface. Thus, it follows from the data of Fig. 4 that a determination of N proceeding solely from the conductive heat exchange of the substrate leads to an error of up to 200% in determining the AFC of the gauge in the investigated range of the dimensionless parameter $\omega d/U$.

It follows from relation (10) that the problem of the inertia of the gauge can be solved by an ordinary method on the assumption that the boundary layer is steady. The AFC thus obtained has to be multiplied by the AFC of the boundary layer which is given by the dependence shown in Fig. 4.

Inertia of the thermal boundary layer does not take place only in film gauges but also in other gauges in which convective heat exchange occurs. However, it has to be determined specifically for each case because according to the theoretical data of [3] it depends on the longitudinal pressure gradient in the gauge, i.e., on its shape.

NOTATION

u, longitudinal velocity component; v, transverse velocity component; x, coordinate in the direction along the plate; y, coordinate transverse to the flow; t, temperature; τ, time; U, velocity of the incoming flow; d, width of the heated film; a, thermal diffusivity of the flowing liquid; δ_t, thickness of the thermal boundary layer; $\omega = 2\pi f$, angular frequency; δ, thickness of the hydrodynamic boundary layer; $Pr = \nu/a$, Prandtl number; ν, kinematic viscosity; $A(\omega)$, amplitude–frequency curve; N, index of thermal inertia of the film gauge; M, time constant of the film; α, heat-transfer coefficient; $\Delta V(f)$ and $\Delta V(0)$, amplitude of the sinusoidal voltage fluctuations in the diagonal of the bridge containing the null indicator with frequencies f and $f \to 0$, respectively; R_g, resistance of the gauge; R_v, resistance of the variable resistor.

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